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Radiative Processes of the DeWitt-Takagi Detector

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Abstract

We examine the excitation of a uniformly accelerated DeWitt-Takagi detector coupled quadratically to a Majorana-Dirac field. We obtain the transition probability from the ground state of the detector and the vacuum state of the field to an excited state with the emission of a Minkowski pair of quanta, in terms of elementary processes of absorption and scattering of Rindler quanta from the Fulling-Davies-Unruh thermal bath in the co-accelerated frame.

Abstract

Examinamos la excitación de un detector de DeWitt-Takagi uniformemente acelerado, acoplado cuadráticamente al campo de Majorana-Dirac. Obtenemos las probabilidades de transición desde el estado base del detector y el vacío del campo al estado excitado con la emisión de un par de cuantos de Minkowski, en términos de procesos elementales de absorción y dispersión de cuantos de Rindler del baño térmico de Fulling-Davies-Unruh en el sistema co-acelerado.

Key words: Classical radiation, Rindler photons, Unruh effect, DeWitt detector

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1 Introduction

The introduction of the concept of particle detectors in quantum field theory in accelerated reference frames and curved spacetimes provides an operational definition of the concept of particle and allows the study of particle production in classical gravitational backgrounds [1],[2].

In this context, the detectors are idealized point objects with internal energy levels which follow a classical trajectory and are coupled to a quantum field. By computing the transition rates of the detector one obtains information of the quantum field from the point of view of non static observers [1],[2], [3]. For a uniformly accelerated detector, this can be done either by quantizing the field in terms of Minkowski modes or working with the alternative quantization procedure [4] using Rindler modes . In the second approach the detector reacts as if immersed in a thermal bath of Rindler particles, which is called Fulling-Davis-Unruh bath.

In a previous work [5] we show that for a scalar field the results of the calculations based on emission/absorption of Rindler quanta in the presence of the Fulling-Davis-Unruh bath give in a simple and physically appealing way the same result of the standard computation involving Minkowski quanta. This was done by analyzing the response function of the detector and is in agreement with previous results using different techniques [6], [7],[8]. In the limit of a zero energy gap detector we reobtained the classical result for the radiation of a charged particle.

In this note we obtain an analogous result for the fermionic case. In particular we show that the response of a DeWitt-Takagi detector prepared in the ground state and uniformly accelerated through the Minkowski vacuum of the massless Dirac field is exactly what one would expect for a static detector immersed in the Fulling-Davies-Unruh thermal bath. In this form we get a direct particle interpretation in terms of elementary processes of inelastic scattering and absorption of Rindler quanta for the emission of a Minkowski pair.

2 Dirac Field in the Rindler Wedge

Let us begin by considering the formulation of a massless Dirac field in Rindler coordinates[9]. The Rindler wedge R_+ is the region $x > |t|$ of Minkowski spacetime, covered by coordinates (ζ, ξ, y, z) which are related to Minkowski coordinates by

$$x = \xi \cosh \zeta \qquad\qquad t = \xi \sinh \zeta. \tag{1}$$

The Minkowski line element in these coordinates is

$$ds^2 = -\xi^2 d\zeta^2 + d\xi^2 + dy^2 + dz^2. \tag{2}$$

Static observers in Rindler coordinates ($\xi = const$) perform hyperbolic motions in Minkowski spacetime with acceleration $a = \xi^{-1}$ and proper time $\tau = \zeta/a$.

In order to write down the solution of the massless Dirac equation

$$\gamma^\mu \nabla_\mu \psi = 0 \tag{3}$$

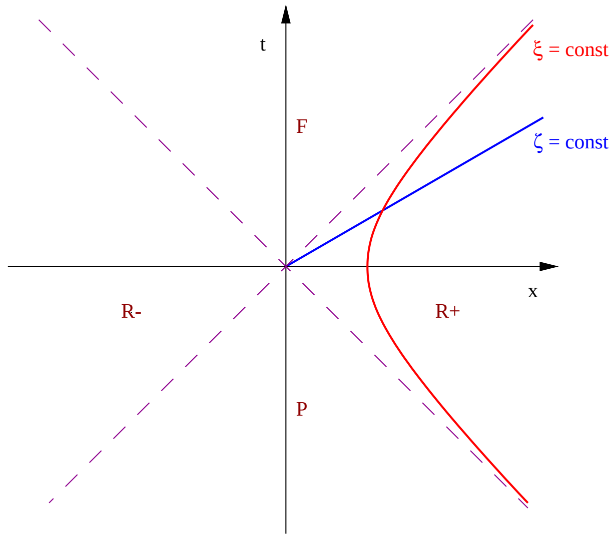

Figure 1: Figure 1. Rindlers partition of Minkowski space

in the Rindler wedge, we follow Candelas and Deutsch approach [10] in the Majorana representation and choose a diagonal vierbein, which gives only one non vanishing component of the spinor connection

$$L^{\mu}_{\alpha} = \mathrm{diag}\,[\frac{1}{\xi}, 1, 1, 1] \qquad\qquad \Gamma_{\zeta} = \frac{1}{2}\gamma^0\gamma^1\,. \tag{4}$$

The positive frequency solutions of the Dirac equation with respect to Rindler time are then given by

$$\psi(x|\nu, \mathbf{k}, \lambda) = \sqrt{\frac{k\cosh \pi\nu}{2\pi^4}} e^{-i\nu\zeta + \mathbf{kx}} K_{i\nu+\frac{1}{2}\gamma^0\gamma^1}(k\xi)\chi(\mathbf{k}, \lambda), \tag{5}$$

in terms of the multicomponent McDonald functions introduced in Ref.[10] and a constant spinor χ. Thus, the field operator may be decomposed as

$$\psi(x) = \sum_{\lambda=\pm} \int_0^{\infty} d\nu \int d^2\mathbf{k}\,\{a(\nu, \mathbf{k}, \lambda)\psi(x|\nu, \mathbf{k}, \lambda) + H.c.\} \tag{6}$$

with the anti-commutation relations on the creation/annihilation operators.

3 DeWitt-Takagi Detector

The conceptually simplest detector one can use to probe the quantum Dirac field, was proposed by Takagi [11], and is obtained by coupling the DeWitt monopole detector to the free Dirac field by means of the interaction lagrangian

$$\mathbf{L}_{int} = m(\tau)\overline{\psi}[x(\tau)]\psi[x(\tau)]. \tag{7}$$

Assuming that the field is initially in the Minkowski vacuum state and the detector in the ground state, the transition probability of the detector to the excited state and to all possible final states of the quantum field is, to lowest order [2],

$$\mathcal{P}(E) = |\langle E|m(0)|0\rangle|^2 \, \mathcal{F}(E), \tag{8}$$

where the response function is given by

$$\mathcal{F}(E) = \sum_{|f\rangle} \left| \int_{-\infty}^{\infty} d\tau e^{iE\tau} \langle f|\overline{\psi}[x(\tau)]\psi[x(\tau)]|0_M\rangle \right|^2 . \tag{9}$$

The response function is proportional to the proper time interval, and one can work with the response function per unit proper time interval $\frac{\mathcal{F}(E)}{T}$ as the relevant physical quantity. To investigate the contributing terms in the amplitude above one first expands ψ in Minkowski modes and let it act on the Minkowski vacuum so that only the creation part of ψ contributes. After that, the action of $\overline{\psi}$ can yield two possible contributions, the reabsorption of the created quantum which can be eliminated by normal ordering of the interaction term, or the creation of another quantum, ending up with a pair of Minkowski quanta in the final state.

Therefore, to lowest order the only possible final state in terms of Minkowski quanta is the one with a pair and the response function per unit proper time interval is proportional the tree level emission rate of a Minkowski pair.

4 Rindler Mode Calculation

The alternative way to handle this computation is to use the Rindler mode expansion of the field. After performing one time integral one gets for the response function $\frac{\mathcal{F}(E)}{T}$

$$\begin{aligned}
&\sum_{|f\rangle}\sum_{J,J'} \delta(E + a\nu' - a\nu) \ \left|\langle f|\, a^\dagger_{J'}\, a_J\, |0_M\rangle\, \overline{\psi}_{J'}(\mathbf{x_0})\, \psi_J(\mathbf{x_0})\right|^2 \\
&\quad + \delta(E - a\nu' - a\nu) \ \left|\langle f|\, a_{J'}\, a_J\, |0_M\rangle\, \overline{\psi}^*_{J'}(\mathbf{x_0})\, \psi_J(\mathbf{x_0})\right|^2 \\
&\quad + \ \delta(E - a\nu' + a\nu) \ \left|\langle f|\, a_{J'}\, a^\dagger_J\, |0_M\rangle\, \overline{\psi}^*_{J'}(\mathbf{x_0})\, \psi^*_J(\mathbf{x_0})\right|^2 ,
\end{aligned} \tag{10}$$

where we are using the compact notation $J \equiv \{\nu, \mathbf{k}, \lambda\}$ and $\psi_J(\mathbf{x_0})$ stands for the spatial part of $\psi(x|\nu, \mathbf{k}, \lambda)$ at $\xi = a^{-1}, y = z = 0$.

Using the completeness of the final states, the fact that the restriction of the Minkowski vacuum state to the Rindler wedge gives the Fermi-Dirac factor for the expectation value of Rindler quanta number and the anti-commutative nature of the fermionic operators, we can identify the contributing terms in the above expression. These are the following: annihilation of a Rindler quantum with frequency $a\nu$ with the creation of another with frequency $a\nu' = a\nu - E$, annihilation of a pair with frequencies $a\nu + a\nu' = E$ and finally, creation of an anti-particle with frequency $a\nu$ and the annihilation of another with frequency $a\nu' = a\nu + E$. In this case the particle and anti-particle are the same, but the same result holds also for the full electron-positron field.

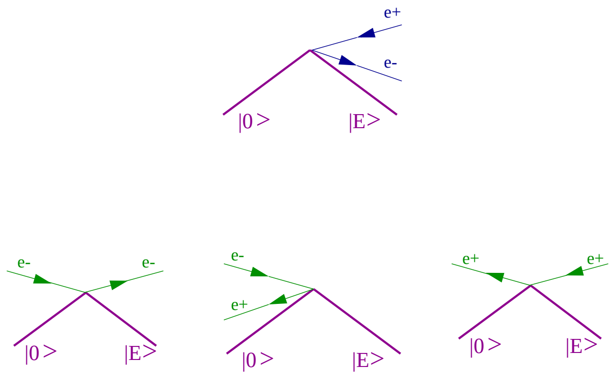

Figure 2: Figure 2. Minkowski pair emission, equivalent to absorption/scattering of Rindler quanta

The tree level emission rate is then obtained in the co-accelerating frame as the contribution of three elementary processes, each one weighted with the appropriate statistical factor and satisfying the energy balance conditions mentioned above

$$\begin{aligned} \frac{d\mathcal{P}_{emis,e_M^- e_M^+}}{d\tau} &= \sum_{J,J'} \left[\frac{1}{e^{2\pi\nu}+1}\right] \left[\frac{1}{e^{2\pi\nu'}+1}\right] \frac{d\mathcal{P}_{absor,e_R^- e_R^+}}{d\tau}\Big|_{J,J'} \\ &+ \left[\frac{1}{e^{2\pi\nu}+1}\right] \left[1-\frac{1}{e^{2\pi\nu'}+1}\right] \frac{d\mathcal{P}_{disp,e_R^-}}{d\tau}\Big|_{J,J'} \\ &+ \left[1-\frac{1}{e^{2\pi\nu}+1}\right] \left[\frac{1}{e^{2\pi\nu'}+1}\right] \frac{d\mathcal{P}_{disp,e_R^+}}{d\tau}\Big|_{J,J'} \end{aligned} \tag{11}$$

There is induced absorption and, in contrast to the bosonic case, the emission is attenuated rather than stimulated. This is a manifestation of Pauli's exclusion principle. It is worth mentioning that a similar attenuation effect was obtained by L. Parker[12] in the context of fermion production in an expanding gravitational background.

A graphical representation of the two different particle interpretations of the 'click' of the DeWitt-Takagi detector is given in fig.2.

We note that Matsas and Vanzella [13], studying the influence of acceleration in particle decays, have proposed to describe the proton and neutron states of the nucleon with a semi-classical current coupled to the electron and neutrino fields. This is a slight modification of the DeWitt-Takagi detector and therefore in the accelerated frame there are also three contributing processes to the proton decay rate and the branching ratios for each channel can be calculated.

5 Conclusion

We discussed the particle interpretation of the behavior of a monopole detector in hyperbolic motion coupled to a fermionic field. For an inertial observer the excitation of the detector comes with the emission of a Minkowski pair of particles at a rate which from the point of view of an accelerated observer corresponds to the combined effects of absorption and scattering of Rindler quanta from the Fulling-Davies-Unruh thermal bath with the

appropriate statistical weights. The presence of fermions in the final state results in an attenuation of the emission rate of Rindler quanta, in agreement with similar effects reported in the literature. The equivalence found here may prove relevant in the analysis of fermion absorption and emission on more general backgrounds, e.g. the exterior region of a black hole.

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